

Non-Fermi-liquid effect in magnetic susceptibility

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Abstract

Taking into account the anomalous self-energy for quarks due to the dynamic screening effect for the transverse gluon propagator, we study the temperature dependence of the magnetic susceptibility in detail. It is shown that there does not exist the $T \ln T$ term in the susceptibility, different from the specific heat, but an anomalous $T^2 \ln T$ term arises instead as a novel non-Fermi-liquid effect.

It is well known that Fermi liquid theory (FLT) is very powerful in discussing the properties of interacting fermions at low temperature [1, 2]. The renormalization-group analysis have shown that the Fermi-liquid theory is a fixed-point theory, where all the quasi-particle interactions are *marginal* [3], except the attractive BCS channel. This argument, however, may be applied to the case of the short-range interaction. Recent renormalization-group (RG) arguments have revealed that the quasi-particle interaction through the exchange of the transverse gauge field is *relevant* and induces the non-Fermi-liquid effects in gauge theories (QED/QCD) [4, 5, 6, 7, 8, 9].

Within FLT, fermions are treated as quasi-particles incorporating the self-energy; the quasi-particle interactions around the Fermi surface are important and physical quantities are given in terms of the Landau-Migdal parameters. In gauge theories (QED/QCD), there appear infrared (IR) singularities in the Landau-Migdal parameters due to their infinite range. To improve the IR behavior in the quasi-particle interaction, it is necessary to take into account the screening effect for the gauge field. Actually, the screening effect have been shown to be important

in the many-body theories [2]; the Coulomb interaction becomes short-ranged by the Debye mass. The inclusion of the screening effect is also required by the argument of the hard-dense-loop (HDL) resummation [10]. Anyhow we can see the static screening by the Debye mass for the longitudinal mode and the IR behavior is surely improved. However, there is no static screening for the transverse mode and there is only the dynamic screening [10]. Thus the IR singularities are still left for the gauge interactions of the transverse mode.

Accordingly the self-energy of the quasi-particles, $\Sigma_+(\epsilon_{\mathbf{k}})$, exhibits an anomalous behavior as $\epsilon_{\mathbf{k}} \rightarrow \mu$ due to the dynamic screening, $\text{Re}\Sigma_+(\epsilon_{\mathbf{k}}) \sim g^2/9\pi^2(\epsilon_{\mathbf{k}} - \mu)\ln(\Lambda/|\epsilon_{\mathbf{k}} - \mu|)$ within the one-loop calculation [8, 11]. Such an anomalous self-energy gives rise to the non-Fermi-liquid behavior in entropy or specific heat [4, 5, 6, 8, 12, 15, 16]. An anomalous contribution to specific heat, $\propto T\ln T$ at low T , has been firstly shown by Holstein et al. in the case of electron gas[12]. It may be understood within FLT that the density of state at the Fermi surface behaves like $\ln T$. Recently the analogous effect has been discovered in QCD [8, 15]. Similar effects due to dynamical gauge fields in systems of strongly correlated electrons were studied in refs. [4, 5, 6, 7].

In this Letter we study the magnetic susceptibility of the gauge theories at finite temperature ¹. The magnetic susceptibility has been one of the important physical quantities within FLT since the original work of Landau, and repeatedly utilized to study the magnetic properties in condensed-matter physics [1, 2]. Recently, the magnetic properties of QCD or its magnetic instability would be an interesting subject [17, 18, 19, 20, 21, 22, 23] in relation to phenomena of compact stars, especially magnetars [24] or primordial magnetic field in early universe, where one may expect the QCD phase transition [25].

In a recent paper we have studied the magnetic susceptibility of quark matter at $T = 0$ within the FLT to figure out the screening effects for gluons on the magnetic instability [26, 27]. We have seen that the transverse gluons still gives logarithmic singularities for the Landau-Migdal parameters, but they cancel each other in the magnetic susceptibility to give a finite result.

At $T \neq 0$, the Fermi surface is smeared over the width $O(T)$, so that the dynamic screening effect should give rise to a logarithmic T dependence for physical quantities. Then, one may expect a similar non-Fermi liquid effect in the magnetic susceptibility as in the specific heat, because both quantities are related to the density of states at the Fermi surface. However, since there does not exist in the liquid

¹Here we only consider QCD, but our results are easily applied to electron gas with small modification.

the close relation between the specific heat and the magnetic susceptibility that exists in gases, we shall see a different non-Fermi-liquid effect. Actually we find that there appears $T^2 \ln T$ term in the magnetic susceptibility as another non-Fermi liquid effect.

In the following we consider the color-symmetric interaction among quasi-particles: it can be written as the sum of two parts, the spin independent ($f_{\mathbf{k},\mathbf{q}}^s$) and dependent ($f_{\mathbf{k},\mathbf{q}}^a$) ones;

$$f_{\mathbf{k}\zeta,\mathbf{q}\zeta'} = f_{\mathbf{k},\mathbf{q}}^s + \zeta\zeta' f_{\mathbf{k},\mathbf{q}}^a. \quad (1)$$

Since quark matter is color singlet as a whole, the Fock exchange interaction gives a leading contribution [17, 18, 20]. We, hereafter, consider the one-gluon-exchange interaction (OGE). Since the OGE interaction is a long-range force and we consider the small energy-momentum transfer between quasi-particles, we must treat the gluon propagator by taking into account the screening effects [10];

$$D_{\mu\nu}(k-q) = P_{\mu\nu}^t D_t(p) + P_{\mu\nu}^l D_l(p) - \xi \frac{P_\mu P_\nu}{p^4} \quad (2)$$

with $p = k - q$, where $D_{t(l)}(p) = (p^2 - \Pi_{t(l)})^{-1}$, and the last term represents the gauge dependence with a parameter ξ . Note that the quasi-particle interaction (1) is independent of the gauge choice [27]. $P_{\mu\nu}^{t(l)}$ is the standard projection operator onto the transverse (longitudinal) mode [10]. Since the soft gluons should give a dominant contribution in our case, we must sum up an infinite series of the polarization functions (the hard-dense-loop (HDL) resummation) in the gluon propagator. HDL resummation then gives the polarization functions for the transverse and longitudinal gluons as

$$\begin{aligned} \Pi_l(p_0, \mathbf{p}) &= \sum_{f=u,d,s} m_{D,f}^2, \\ \Pi_t(p_0, \mathbf{p}) &= -i \sum_{f=u,d,s} \frac{\pi u_{F,f} m_{D,f}^2}{4} \frac{p_0}{|\mathbf{p}|}, \end{aligned} \quad (3)$$

in the limit $p_0/|\mathbf{p}| \rightarrow 0$, with $u_{F,f} \equiv k_{F,f}/E_{F,f}$ and the Debye mass for each flavor, $m_{D,f}^2 \equiv g^2 \mu_f k_{F,f} / 2\pi^2$ [10]². Thus the longitudinal gluons are statically screened to have the Debye mass, while the transverse gluons are dynamically screened by the Landau damping. Accordingly, the screening effect for the transverse gluons is ineffective at $T = 0$, where soft gluons contribute. At finite temperature, gluons

²The Debye mass is given as $e^2 \mu^2 u_F / \pi^2$ for electron gas in QED.

with $p_0 \sim O(T)$ can contribute due to the diffuseness of the Fermi surface and the transverse gluons are effectively screened.

We consider the magnetic susceptibility at low temperature. We, hereafter, concentrate on one flavor and omit the flavor indices for simplicity. Magnetic susceptibility is then written in terms of the quasi-particle interaction [1, 2, 27],

$$\chi_M = \left(\frac{\bar{g}_D \mu_q}{2} \right)^2 \frac{1}{N^{-1}(T) + \bar{f}^a} \quad (4)$$

where \bar{g}_D is the gyromagnetic ratio [27]. $N(T)$ is an extension of the density of state at the Fermi surface for $T \neq 0$;

$$N(T) = -2N_c \int \frac{d^3k}{(2\pi)^3} \frac{\partial n(\epsilon_{\mathbf{k}})}{\partial \epsilon_{\mathbf{k}}} \quad (5)$$

with the Fermi-Dirac distribution function, $n(\epsilon_{\mathbf{k}}) \equiv (1 + e^{\beta(\epsilon_{\mathbf{k}} - \mu)})^{-1}$, where $\epsilon_{\mathbf{k}}$ is the quasi-particle energy. At $T = 0$ we have $N(0) = (N_c k_F^2 / \pi^2) v_F^{-1}$ with the Fermi velocity, $v_F = k_F / \mu - (N_c k_F^2 / 3\pi^2) f_1^s$ in terms of the Landau-Migdal parameter f_1^s [28].

\bar{f}^a is a spin-dependent Landau-Migdal parameter given by

$$\bar{f}^a \equiv -2N_c \int \frac{d^3k}{(2\pi)^3} \frac{\partial n(\epsilon_{\mathbf{k}})}{\partial \epsilon_{\mathbf{k}}} \int \frac{d\Omega_{\mathbf{q}}}{4\pi} f_{\mathbf{k}, \mathbf{q}}^a \big|_{|\mathbf{q}|=k_s} / N(T), \quad (6)$$

where k_s is defined by $\epsilon_{k_s} = \mu$ and coincides with the usual Fermi momentum k_F at $T = 0$.

Note that in both Eqs. (5) and (6) the function, $-\partial n(\epsilon_{\mathbf{k}}) / \partial \epsilon_{\mathbf{k}}$, is sharply peaked at $\epsilon_{\mathbf{k}} = \mu$ for $T/\mu \ll 1$, and we can see that only the quasi-particles near the Fermi surface still gives a dominant contribution. However, we cannot use the standard low-temperature expansion, since the quasi-particle energy is not regular at the Fermi surface due to the no screening for transverse gluons.

First we study the average of the density of state given by Eq. (5). We can rewrite it as

$$\begin{aligned} N(T) &= \frac{N_c}{\pi^2} \int_{\epsilon_0}^{\infty} d\omega \frac{dk}{d\omega} k^2 \frac{\beta e^{\beta(\omega - \mu)}}{(e^{\beta(\omega - \mu)} + 1)^2} \\ &\simeq \frac{N_c}{\pi^2} \int_{\epsilon_0}^{\infty} d\omega \left(1 - \frac{\partial \text{Re} \Sigma_+(\omega)}{\partial \omega} \right) k(\omega) E_{k(\omega)} \frac{\beta e^{\beta(\omega - \mu)}}{(e^{\beta(\omega - \mu)} + 1)^2}, \quad (7) \end{aligned}$$

with $\varepsilon_0 \equiv \varepsilon_{|\mathbf{k}|=0}$, where ω is the quasi-particle energy and $k(\omega)$ satisfies

$$\omega = E_{k(\omega)} + \text{Re}\Sigma_+(\omega, k(\omega)). \quad (8)$$

The one-loop self-energy is almost independent of the momentum³, and can be written as [8, 11]

$$\begin{aligned} \text{Re}\Sigma_+(\omega, k) \sim \text{Re}\Sigma_+(\mu, k_F) - \frac{C_f g^2 u_F}{12\pi^2} (\omega - \mu) \ln \frac{\Lambda}{|\omega - \mu|} \\ + \Delta^{\text{reg}}(\omega - \mu) \end{aligned} \quad (9)$$

around $\omega \sim \mu$ with $C_f = (N_c^2 - 1)/(2N_c)$ and $u_F = k_F/E_{k_F}$. Λ is a cut-off factor and should be taken as an order of the Debye mass, $\Lambda \sim M_D \equiv \left(\sum_f m_{D,f}^2\right)^{1/2}$. The self-energy has an imaginary part, $\text{Im}\Sigma_+(\omega, k) \sim C_f g^2/24\pi|\omega - \mu|$, which measures the life time for quasi-particles. In the following we only use the real part, since we are interested in quasi-particle near the Fermi surface. Note that the anomalous term in Eq. (9) appears from the dynamic screening of the transverse gluons, while the contribution by the longitudinal gluons is summarized in the regular function $\Delta^{\text{reg}}(\omega - \mu)$ of $O(g^2)$. The longitudinal gluons then gives $O(g^2 T^2)$ contribution to $N(T)$ as in the usual situation. Thus the leading-order contribution comes from the transverse gluons. We, hereafter, extract only the transverse contribution, $N_t(T)$, using the anomalous term in Eq. (9): substituting Eq. (9) into Eq. (7), we obtain

$$\begin{aligned} N_t(T) = \frac{N_c k_s \mu}{\pi^2} \left[1 + \frac{\pi^2}{6} \frac{(2k_F^2 - m^2)}{k_F^4} T^2 \right. \\ \left. + \frac{C_f g^2 u_F}{24} \frac{(2k_F^2 - m^2)}{k_F^4} T^2 \ln \left(\frac{\Lambda}{T} \right) + \frac{C_f g^2 u_F}{12\pi^2} \ln \left(\frac{\Lambda}{T} \right) \right] + O(g^2 T^2). \end{aligned} \quad (10)$$

$N_t(T)$ or its inverse, $N_t^{-1}(T)$, has a term proportional to $\ln T$ and gives the leading-order contribution. It has a singularity at $T = 0$, which corresponds to the logarithmic divergence of the Landau-Migdal parameter f_1^s at $T = 0$ [27, 29]. We have kept the next-to-leading order term ($T^2 \ln T$ term) in Eq. (10), because we shall see that the $\ln T$ term is canceled out by another term appearing in the spin-dependent Landau-Migdal parameter \bar{f}^a in the magnetic susceptibility⁴.

³A renormalization group argument indicates that theory is infrared free in this case and the solution of the Schwinger-Dyson equation is almost the same as the one-loop result [9].

⁴This is a different feature from the specific heat, where $\ln T$ term remains in the final result as the leading-order contribution.

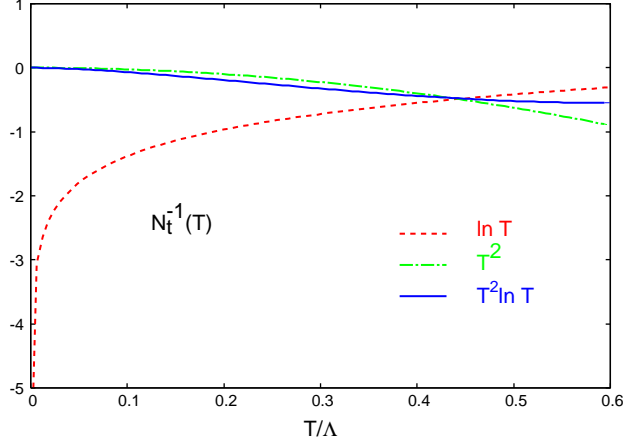


Figure 1: Schematic view of each contribution to $N_t^{-1}(T)$. The leading order contribution ($\ln T$) is canceled in the magnetic susceptibility, so that the next-to-leading order contribution ($T^2 \ln T$) becomes dominant.

There are two contributions to \bar{f}^a : one is given by the longitudinal mode, \bar{f}_l^a , and the other by the transverse mode, \bar{f}_t^a . The transverse component \bar{f}_t^a has a logarithmic singularity at $T = 0$ due to the absence of the static screening. On the other hand, the longitudinal component \bar{f}_l^a has no IR singularity because of the static screening, and is almost temperature independent. Thus the leading-order contribution at finite temperature comes from the transverse gluons again as for $N(T)$. \bar{f}_t^a is given by

$$\bar{f}_t^a = 2N_c N^{-1}(T) \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\Omega_{\mathbf{q}}}{4\pi} \frac{\partial n(\epsilon_{\mathbf{k}})}{\partial \epsilon_{\mathbf{k}}} \frac{m^2}{E_s E_{\mathbf{k}}} C_f N_c^{-1} g^2 M^{ia}(\mathbf{k}, \mathbf{q}) D_t(k-q) \Big|_{|\mathbf{q}|=k_s} \quad (11)$$

where M^{ia} is the reduced matrix element for the spin-dependent interaction [27], and we defined E_s by $E_s = E_{|\mathbf{q}|=k_s}$. It is the dynamic screening part in the propagator D_t that gives the $\ln T$ -dependence to \bar{f}_t^a . Therefore, we can put $|\mathbf{k}| = k_s$ in the other parts of the integrand in Eq. (11). The real part of the transverse propagator then render

$$\text{Re} D_t(k-q) \Big|_{|\mathbf{q}|=k_s} \simeq -\frac{1}{2k_s^2} \frac{(1 - \cos \theta_{\widehat{\mathbf{kq}}})^2}{(1 - \cos \theta_{\widehat{\mathbf{kq}}})^3 + c^3 (E_{\mathbf{k}} - E_s)^2} \quad (12)$$

with $c^3 \equiv \frac{1}{8k_s^6} \sum_f \left(\frac{\pi m_{D,f}^2 u_{F,f}}{4} \right)^2$, while the imaginary part gives only higher order terms with respect to temperature and thus we neglect it here.

For low T , the angular integrals in Eq. (11) give a $\ln T$ dependence;

$$\begin{aligned} \bar{f}_t^a &\simeq -\frac{C_f g^2}{12\pi^2 E_s^2 T} N^{-1}(T) \int_{\varepsilon_0}^{\infty} d\omega k(\omega) E_{k(\omega)} \left(1 - \frac{\partial \text{Re}\Sigma_+(\omega)}{\partial \omega} \right) \ln(|E_{k(\omega)} - E_s|) \frac{\partial n(\omega)}{\partial \omega} \\ &\sim -\frac{C_f g^2}{12N_c \mu^2} \ln T, \end{aligned} \quad (13)$$

where the term $\partial \text{Re}\Sigma_+(\omega)/\partial \omega$ in the integrand does not contribute up to $O(g^2)$ in this calculation. Note that there appears no T^2 or $T^2 \ln T$ term in the Landau-Migdal parameter.

Comparing Eq. (13) with Eq. (10), one can see that the $\ln T$ terms, which are the leading-order contribution, exactly cancel each other in the magnetic susceptibility (4) through the combination, $N_t^{-1}(T) + \bar{f}_t^a$. Thus the remaining temperature dependent terms in the magnetic susceptibility have $T^2 \ln T$ terms as next-to-leading order (NLO) contribution, as well as usual T^2 terms (see Fig. 1).

It is important to remember that the chemical potential is temperature dependent as well and other temperature dependence comes in the magnetic susceptibility. The temperature dependence of the chemical potential can be derived by considering the temperature variation on the quasi-particle number density ρ , $d\rho/dT = 0$ [15],

$$\mu(T) = \mu_0 - \frac{\pi^2}{6} \frac{(2k_F^2 + m^2)}{k_F^2 E_F} T^2 \left(1 + \frac{C_f g^2}{12\pi^2} \ln \left(\frac{\Lambda}{T} \right) \right) + O(g^2 T^2). \quad (14)$$

It would be interesting to see that the chemical potential has $T^2 \ln T$ term besides the usual T^2 term due to the transverse gluons. Taking into account this temperature-dependence in Eqs. (10) and (13), we finally find the temperature dependent part of the magnetic susceptibility $\delta\chi_M$,

$$\begin{aligned} \delta\chi_M^{-1} = \chi_{\text{Pauli}}^{-1} &\left[\frac{\pi^2}{6k_F^4} \left(2E_F^2 - m^2 + \frac{m^4}{E_F^2} \right) T^2 \right. \\ &\left. + \frac{C_f g^2 u_F}{72k_F^4 E_F^2} (2k_F^4 + k_F^2 m^2 + m^4) T^2 \ln \left(\frac{\Lambda}{T} \right) \right] + O(g^2 T^2), \end{aligned} \quad (15)$$

where χ_{Pauli} is the Pauli paramagnetism, $\chi_{\text{Pauli}} \equiv \bar{g}_D^2 \mu_q^2 N_c k_F \mu / 4\pi^2$. It is evident that there appears $T^2 \ln T$ dependence in the susceptibility at finite temperature

besides the usual T^2 dependence. This corresponds to $T \ln T$ term in the specific heat [4, 5, 6, 8, 12, 15, 16] and is a novel *non-Fermi-liquid effect* in the magnetic susceptibility. At low temperature, $\ln(\Lambda/T) > 0$ so that the $T^2 \ln T$ term gives positive contribution to χ_M^{-1} . Therefore, both T -dependent terms in Eq. (15) work against the magnetic instability, which is characterized by the condition, $\chi_M \rightarrow 0$.

We have discussed the non-Fermi-liquid effect in the magnetic susceptibility for gauge theories (QED/QCD), where the screening effects for gluons are properly taken into account. Since the quasi-particle energy is not regular on the Fermi surface, we cannot use the low temperature expansion, different from the usual treatment in FLT. Carefully extracting the temperature dependence, we have found that the interesting features of the magnetic properties in gauge theories, especially an anomalous $T^2 \ln T$ contribution to the magnetic susceptibility by the transverse gluons. It may be interesting to recall that the static screening gives the magnetic susceptibility the term proportional to $M_D^2 \ln M_D^{-1}$ at $T = 0$ [27]; the Debye mass M_D works as an infrared (IR) cutoff to in the quasi-particle interaction due to the longitudinal gluons, while there is no static screening for the transverse gluons. At finite temperature, the Fermi surface is smeared over order T , so that temperature itself plays a role of the IR cutoff through the dynamic screening in the quasi-particle interaction due to the transverse gluons.

The logarithmic temperature dependence appears in the magnetic susceptibility as a novel non-Fermi-liquid effect, and its origin is the same as in the well-known $T \ln T$ dependence of the specific heat [4, 5, 6, 8, 12, 15, 16]. However, recall that there is no relation between the specific heat and the magnetic susceptibility within FLT, different from gases. Actually we have seen that the $\ln T$ term in $N(T)$ is exactly canceled by the spin-dependent interaction to leave $T^2 \ln T$ term as a leading-order contribution. The anomalous $T^2 \ln T$ term works against the magnetic instability, as well as the usual T^2 term.

We shall report elsewhere the consequences of the non-Fermi-liquid effect in more detail and the magnetic phase diagram in QCD on the density-temperature plane[29].

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